Statistical model calculations in heavy ion reactions

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Results of various fusion experiments with heavy ions are compared with predictions of statistical model calculations. In some reactions there is evidence for nonstatistical effects based on significant discrepancies between the calculations and the experimental results. Alternative explanations of these discrepancies are considered.

[NUCLEAR REACTIONS HI (xnγ), statistical model calculations.]

INTRODUCTION

The advent of heavy ion accelerators has facilitated the study of nuclei at high excitation energy and angular momentum. Measurements of the gamma-ray multiplicity and evaporation residue cross sections show that there is a linear correlation between the average gamma-ray multiplicity and the maximum angular momentum of the evaporation residues. It has been demonstrated that the multiplicity is apparently limited by a critical angular momentum (denoted ) at which the rotating liquid drop fission barrier falls below the neutron binding energy.

It has generally been assumed that most of the decay properties of the excited nuclei produced in these reactions can be described by statistical model calculations, although no such detailed calculations spanning all available experimental results have been reported yet. In spite of the lack of comprehensive calculations, some properties of these reactions have been ascribed to non-equilibrium effects.

These are (1) the persistence of significant cross sections with low at high excitation energies, and (2) overlapping gamma-ray multiplicity distributions for different channels. This approach has been challenged by Blinn and Ferguson, who assert the necessity for performing complete evaporation calculations prior to deducing the existence of nonstatistical effects.

The purpose of this study is to examine the extent to which all available data (in the mass 140–170 region) can be interpreted in terms of evaporation calculations using a Fermi gas level density parametrization. The reactions considered are Ar + Ag, Ar + Sn, Ar + Sn, Ne + Na, and Ar + Te. We find significant discrepancies between our calculations and the experimental results in the Ar + Sn data (mainly at 236 MeV) and in the Ar + Sn data above 210 MeV, which could be due to nonstatistical effects. The possibility of interpreting these data within the framework of statistical model calculations is considered.

The data we attempt to reproduce in these calculations are (1) the relative cross sections for xn (and some xα) reactions, (2) the average angular momentum leading to specific xn channels as determined by multiplicity measurements, (3) the critical angular momentum limiting the survival of evaporation residues.

METHOD OF CALCULATION

The process of the deexcitation of the excited nuclei was calculated using a modified version of code JULIAN which follows the correct procedure for angular momentum coupling at each stage of deexcitation.

For any specific bombarding energy, the partial cross section for compound nucleus formations at angular momentum is

where is the reduced wavelength, and is taken to be

where is a diffuseness parameter and is determined by the total fusion cross section since

The transmission coefficients for light particle emission were determined using optical model potentials of Refs. 15 and 16. For , fission can compete with particle emission.
some of the experiments reported only the evaporation residue (ER) cross section has been measured. In these cases the input fusion cross section was calculated using the method of Bass.\textsuperscript{17} The ER cross section is then determined by two other parameters: (1) the ratio of level densities at the saddle point and at the ground state, (2) the height of the fission barrier (which depends on the total spin).\textsuperscript{9}

The deexcitation process is followed by a Monte Carlo procedure; the results presented were obtained using a thousand deexcitation cascades (events). Several important modifications were made to the code in order to adapt it to the problems under consideration and to shorten its running time: (1) Transmission coefficients for light particle ($n, p, \alpha$) evaporation are obtained during the first step of deexcitation by a full optical model calculation. In subsequent stages of deexcitation the coefficients are obtained by extrapolation from the initial ones. (2) A fission decay mode was added using a rotating liquid drop fission barrier routine.\textsuperscript{14} (3) Angular momentum projections are calculated at each stage of deexcitation—this enables the determination of the angular distribution of the emitted particles. (4) A traceback feature has been included enabling determination of the decay chains and region of the $E-J$ plane leading to specific nuclei. In addition we have introduced a dispersion of the initial excitation energy to account for target thickness effects.

The level density $\rho(E, J)$ used in these calculations above ~5 MeV is given by

$$\rho(E, J) = \rho_0(U)(2J + 1) \exp\left[2\left[a(U - E_{\text{ref}}(J))\right]^{1/2}\right].$$

(1) The persistence of $\alpha n$ cross sections with low $x$ at high excitation energies advocates a high fission barrier and low value of $a_f/a$—this enables the high spin channels to decay by neutron emission (via a small number of neutrons due to the proximity of the yrast line). (2) Partial waves above $l = 65$ tend predominantly to fission.\textsuperscript{9} This second condition is compatible with $a_f/a = 1.05$ and a fission barrier height which is 10-20\% lower than the rotating liquid drop barrier. The compromise we have chosen is to attempt to fit all the data with $a_f/a = 1.00$ and a fission barrier which is 0.8 times the rotating liquid drop barrier.

Figure 1 compares calculated results of $I_{\text{crit}}$ compared to experimental values obtained from gamma-ray multiplicity measurements and from ER cross-section measurements using the sharp cutoff approximation. The lower line is the fit to the $\text{Ar}^{198}\text{Ag}$ data and the upper line—the $\text{Ar}^{194}\text{Sn}$ data. The calculations predict a critical angular momentum for ER survival of $I_{\text{crit}} = 70$, which is within the published errors of the various data. A detailed comparison of ER and fission cross sections for the $\text{Ar}^{198}\text{Ag}$ reaction (the total fusion cross section is used as input) is presented in Table I. The $l$-diffuseness parameter $\Delta$ has very little effect on the results except at 169 MeV where the high-$l$ tail of $\sigma_l$ determines the fission cross section; $\Delta = 5.0$ results in a 6 mb fission cross section at this energy.

### B. $\alpha n$ and $\alpha x$ cross sections

All calculations presented in this section use fusion cross sections obtained from the Bass

![FIG. 1. Maximum angular momentum of ER($l_{\text{max}}$) obtained from calculations (full line) and experiment. The upper line is for the $^{152}$Yb compound nucleus and the lower—$^{148}$Tb. The full points are gamma multiplicity measurements ($l = 2.3M - 2, l_{\text{max}} = 2$) and the hollow points—from cross-section measurements ($l = 3.5l(\Delta, \text{crit} + 1)$). ○—Ref. 1, ▽—Ref. 3, △—Ref. 4, ○—Ref. 6.](image)
TABLE I. Comparison between experimental and calculated evaporation residue ($\sigma_{ER}$) and fission ($\sigma_{fim}$) cross sections in the Ar+Ag reaction. The experimental fusion cross section ($\sigma_{fus}$) is used as input to the calculations to determine the partial wave distribution (see text). At 169 MeV a value of $\Delta = 5.0$ is necessary to obtain the calculated fission cross section. At higher energies, the calculated cross sections are insensitive to $\Delta$. All energies are in MeV and cross sections in mb.

<table>
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<th>$\sigma_{fim}$</th>
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<td>435 ± 70</td>
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<td>1170</td>
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<td>169</td>
<td>1270</td>
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TABLE II. Comparison between experimental and calculated $xn$ and $x\nu\gamma$ cross sections. The experimental $9n$ cross sections of Ref. 4 have been multiplied by a factor of 2 according to Ref. 10. The $6\nu\gamma$ cross sections are included in the $6n$ cross sections. The input fusion cross sections were obtained from the Bass model (Ref. 17) for 144, 163, and 172 MeV. At 128 MeV the experimental $xn + x\nu\gamma$ cross section was taken as the fusion cross section.

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<th>$9n$</th>
<th>$10n$</th>
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<th>$6\nu\gamma$</th>
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TABLE III. Same as Table II for Ar+Te reactions (Ref. 1). $\sigma_{fus}$ was taken from the Bass model (Ref. 17).

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<th>Ar+ $^{125}$Te (157 MeV)</th>
<th>Ar+ $^{125}$Te (181 MeV)</th>
<th>Ar+ $^{130}$Te (181 MeV)</th>
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<td>7n</td>
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<tr>
<td>$\sigma_{fus}$</td>
<td>300</td>
<td>767</td>
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model.\textsuperscript{15} Table II presents a comparison between calculations and experimentation between $\pi n$ and $\pi p$ cross sections in the Ne$^+$\textsuperscript{152}Nd reaction. Most of the calculated values come reasonably close to the experimental results. Some notable exceptions are the $\pi n$ cross section at 128 MeV and the $\gamma n$ and $10\omega n$ cross sections at 172 MeV. The $\gamma n$ cross-section discrepancy could be due to nonequilibrium effects (see below); we have no explanation for the other discrepancies. The comparison presented in Table III for Ar$^+$\textsuperscript{126}Te and Ar$^+$\textsuperscript{130}Te generally shows good agreement between experimental and calculated cross sections.

Significant systematic discrepancies appear in the $\pi n$ and $\pi p$ cross sections of the Ar$^+$\textsuperscript{116}Sn reaction,\textsuperscript{7} at beam energies above 200 MeV (see Fig. 2). The experimental cross sections decrease more slowly at high bombarding energies compared to the calculated values. A similar discrepancy can be observed in the Ar$^+$\textsuperscript{124}Sn data\textsuperscript{9} at 236 MeV: Considering the comparison in Table IV, the calculated $\pi n$ cross section falls below the experimental value. These discrepancies remain even if we raise the fission barrier so that partial waves up to $l_{exp}=80$ can decay by particle emission, and use $\Delta=0.5$ which also increases the ER cross section at the expense of the fission channels. Most of the cross section at high angular momenta (at these excitation energies) goes into $\pi n\pi$ reactions. Hillis \textit{et al.}\textsuperscript{3} have deduced from these data (and from gamma-ray measurements—see below) that nonequilibrium effects are responsible for the discrepancies.

C. Gamma-ray multiplicities

The correlation between the ER cross section and the average gamma-ray multiplicity $M_\gamma$ has been investigated in detail in Ref. 1. Assuming $\sigma_{\gamma n}$ and the partial $\pi n$ cross sections to be determined by the sharp cutoff model, they deduce the correlation between the average angular momentum $I$ and $M_\gamma$ to be

$$I = f(M_\gamma - 2)$$

with $f=2.3$ and $\delta=4$. The factor of 2.3 (rather than 2.0) is interpreted as accounting for angular momentum removed by particle emission and statistical gamma rays.\textsuperscript{1} We have reanalyzed various multiplicity data\textsuperscript{1-4} using the average angular momentum leading to specific $\pi n$ channels, $I$, as determined by our calculations. Figure 3 shows the experimental multiplicities $M_\gamma$ (Refs. 1 and 4) vs the pre-evaporation average angular momentum $I$. The line drawn is $I=2.3(M_\gamma - 2)$. Figure 4 shows the same results for the average angular momentum $I_\gamma$ after particle emission. The line drawn is $I_\gamma = 2(M_\gamma - 2)$. Our value of $\delta$ is different from that of Simon \textit{et al.},\textsuperscript{1} who conjectured that out of the total $M_\gamma$ photons, four are statistical $E1$ photons which do not remove any angular momentum; thus $\delta=4$. We find that on average 0.6 units of angular momentum are removed by the statistical gamma transitions. Thus we expect

$$I_\gamma = 2(M_\gamma - 4) + 4 \times 0.6 = 2(M_\gamma - 2.8)$$

which is within two units of the angular momenta we obtain.

We conclude that our calculations can successfully correlate pre-evaporation and post-evaporation angular momenta, about 15% if the initial average angular momentum is removed by evaporated neutrons in the excitation energy range we consider here. The actual amount of angular momentum removed is plotted in Fig. 5: The more angular momentum brought in by the reaction and the less neutrons emitted—the larger the angular momentum removed by a single neutron.
TABLE IV. Comparison between experimental and calculated values for $\text{Ar} + ^{124}\text{Sn}$ of (1) percent of total $xn$ cross section, (2) average kinetic energy of neutrons in specific $xn$ channels, (3) ER cross sections. The experimental cross sections were determined using the sharp cutoff approximation with the correlation of Ref. 1: $T = 2.3 \left( M_x - 2 \right)$, $I_{\text{max}} = 3/2T$, $\sigma_{\text{ER}} = \pi \lambda^2 (I_{\text{max}} + 1)^2$.

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<th>$xn$</th>
<th>% $\Sigma n$</th>
<th>% $\Sigma n$</th>
<th>$\epsilon_n$</th>
<th>$\epsilon_n$</th>
<th>$\sigma_{\text{ER}}$</th>
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DISCUSSION

We have attempted to interpret experimental data in a variety of heavy ion reactions using a simple level density parametrization and transmission coefficients obtained from standard optical model potentials. More realistic calculations should consider the dependence of the level density and transmission coefficients on the nuclear shape at different angular momenta—at large angular momenta, oblate or triaxial shapes may be present. Level densities for these deformed nuclear shapes can be determined by methods such as that.

**FIG. 3.** Gamma multiplicity as a function of initial average angular momentum in $xn$ reactions. Diamonds—multiplicity data of Ref. 1. Triangles—data of Ref. 4. Line $I = 2.3(M - 2)$.

**FIG. 4.** Gamma multiplicity as function of angular momentum $I_{\gamma}$ after particle evaporation. Diamonds—data Ref. 1. Triangles—data of Ref. 4. The full triangles are $x\alpha$ multiplicity data. Line is $I_{\gamma} = 2(M - 2)$. 
of Moretto\textsuperscript{20} using the single particle levels at the deformation appropriate to angular momentum.\textsuperscript{19} Collective enhancement factors resulting from the breaking of various rotational symmetries should also be incorporated.\textsuperscript{21} A first attempt at such a calculation for reactions leading to $A = 56$ was performed by Canty, Gottschalk, and Pühloher\textsuperscript{22}; quadrupole deformations were included in their calculations using Nilsson or Woods–Saxon potentials to generate the single particle level scheme. They generally obtained good agreement between measured and calculated cross sections, although for some residual nuclei there are discrepancies of up to a factor of 2.

Excluding Ar + Sn reactions a few discrepancies are evident in our calculations for some residual nuclei. We now consider the systematic discrepancies in the Ar + Sn reactions (Table IV and Fig. 2): In addition to the tendency of the calculations to underestimate the $x\pi$ partial cross sections with relatively small $\chi$ (for the highest bombarding energies), Hillis et al.\textsuperscript{3} find overlapping gamma multiplicity distributions for the different $\chi\pi$ channels. This is reminiscent of results obtained in the $^{12}$C + $^{149}$Gd reaction\textsuperscript{15,16} above 142 MeV (where nonequilibrium neutron emission has been observed directly) and in $^{16}$O + $^{144}$Nd at 115 MeV.\textsuperscript{23}

There are two reasons why one should consider the possibility that these discrepancies do not result from nonequilibrium processes: First, the energy per nucleon above the Coulomb barrier for the Ar induced reaction is significantly lower than that for the $^{12}$C and $^{16}$O reactions.\textsuperscript{15,23} Second, the similarity between the Ar and Kr induced excitation functions\textsuperscript{7} shows that the $x\pi$ relative cross sections are independent of the mode of formation; thus it would be rather surprising if nonequilibrium effects were responsible for a significant part of these cross sections.

A possible explanation within the statistical model framework could be a change of shape around $J = 60$.\textsuperscript{19} If at this angular momentum (which we denote $J^*$) the nuclear shape were to change from spheroidal to triaxial, the level density for $J > J^*$ would be enhanced by an order of magnitude.\textsuperscript{21} In our present calculations, each neutron in a cascade originating from a spin around $J_1$ takes about 2–3h angular momentum. A level density enhancement above $J > 60$ would cause these neutrons to take less angular momentum and hit the yrast line at a higher energy. This would increase the gamma multiplicity at the expense of the number of neutrons emitted. However, the existence of high spin isomers in the even mass erbium compound nuclei\textsuperscript{24} could be responsible for the low measured gamma multiplicity when the experimental coincidence resolution time is short or comparable to the isomer half-life.

In order to decide which of these possibilities is the correct physical explanation, the spectrum and angular distribution of the emitted neutrons should be measured. Alternatively, investigation of the existence of high spin isomers in specific $x\pi$ channels could indicate which of the possibilities should be accepted.

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\textsuperscript{6}H. C. Britt, B. H. Erkkiela, R. H. Stokes, H. H. Outbrod,
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