Modified empirical parametrization of fragmentation cross sections

K. Sümmerer1,* and B. Blank2,†

1Gesellschaft für Schwerionenforschung, Planckstrasse 1, D-64291 Darmstadt, Germany
2CEN Bordeaux-Gradignan, Le Haut-Vigneau, F-33175 Gradignan Cedex, France

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New experimental data obtained mainly at the GSI/FRS facility allow one to modify the empirical parametrization of fragmentation cross sections. It will be shown that minor modifications of the parameters lead to a much better reproduction of measured cross sections. The most significant changes refer to the description of fragmentation yields close to the projectile and of the memory effect of neutron-deficient projectiles.

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I. INTRODUCTION

The pioneering experiments of projectile fragmentation at relativistic energies of $^{40}\text{Ar}$ and $^{48}\text{Ca}$ beams at the LBL Bevalac [1,2] have demonstrated the potential of this method for the production of exotic nuclei. Based on these ideas, the SIS/FRS facility [3] at GSI has used also heavier projectiles like, e.g., $^{58}\text{Ni}$, $^{80}\text{Kr}$, $^{129}\text{Xe}$, and $^{208}\text{Pb}$ to produce and study exotic nuclei [4–7]. For planning such experiments, when count-rate predictions are needed, analytical descriptions of fragmentation cross sections are useful. They are also useful in simulation programs for projectile-fragment separators (like, e.g., INTENSITY [8] or MOCADI [9]). Compared to physical models of high-energy fragmentation reactions, which in general involve time-consuming Monte Carlo calculations, the virtue of an analytical formula lies in the short computing time and the possibility to calculate easily submicrobarn cross sections that are beyond the reach of physical-model calculations.

In 1990, Sümmerer et al. [10] proposed a universal empirical parametrization of fragmentation cross sections (EPAX [10]) which was based on and similar to previous prescriptions by Rudstam [11] and Silberberg and Tsao [12]. The parametrization was to a large extent based on multi-GeV proton-induced spallation cross sections, since only scarce heavy-ion-induced experimental data were available at that time. Meanwhile, more precise data from relativistic heavy-ion-induced fragmentation reactions together with recent results from projectile fragmentation of heavy nuclei ($^{197}\text{Au}$ and $^{208}\text{Pb}$) on $\text{H}_2$ targets [13,14]) allow a more stringent comparison of proton- and heavy-ion-induced isotope distributions. This comparison indicates that for heavy nuclei the two reactions lead to different isotopic distributions, which cannot be obtained from each other just by scaling factors. This can be easily understood since heavy-ion-induced reactions are expected to deposit more excitation energy in a nucleus than proton-induced reactions, making the final product distributions — after evaporation — broader and more neutron deficient. Nevertheless, the data show that in both cases the isotopic yield distributions can be well described by Gaussian-like analytical functions with parameters that vary smoothly as a function of fragment mass [10]. In the present paper, we will base the choice of these parameters exclusively on heavy-ion-induced reaction data.

We will first review briefly the basic characteristics of the EPAX formula and then show which modifications are necessary to improve the accuracy with which the new experimental results can be reproduced. This will be followed by a brief comparison with similar attempts by other authors.

II. EPAX FORMULA

A. Basic characteristics

The basic characteristics of the analytical description of high-energy fragmentation cross sections by the EPAX formula are the following [10].

(i) In the absence of systematic excitation-function measurements of heavy-ion-induced fragmentation reactions, the formula is valid only for the so-called “limiting fragmentation” regime, i.e., for projectile energies where the fragmentation yields are no longer energy dependent, at least within the accuracy of the formula (approximately within a factor of 2). This is certainly true for incident energies considerably above the Fermi energy in nuclei ($\sim 40\text{ A MeV}$), in particular for the typical SIS energies of (500–1000)A MeV.

(ii) The EPAX formula is meant to describe the fragmentation of medium- to heavy-mass projectiles; nucleon-pickup cross sections are not included. No attempt is made to describe the fragmentation of fissionable nuclei. Therefore, the range of validity is limited to projectiles from around argon below U where fission competition is significant require an elaborate description of the fission process, such as can be found, e.g., in a recent publication by Benlliure et al. [15].

(iii) For fragments sufficiently far away from the projectile (i.e., for mass losses larger than 15–20 % of the projectile mass), the isotope distributions are largely independent of the original nucleus; their position, shape, and width depend only on the fragment mass number. This constitutes what has been termed the “residue corridor” and is related to the fact that the isotope distributions are mainly governed by statistical evaporation from highly excited prefragments produced in collisions between relativistic heavy ions.

(iv) For fragments that involve only a small mass loss from the projectile, the isotope distributions should be cen-
TABLE I. Constants used in the EPAX formula version 1 (Ref. [10]) and those used in version 2 (this work). Note that in a few cases also the functional form to calculate a parameter has changed (see text).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Constant</th>
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<th>Version 2</th>
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terred close to the projectile and their variance should be small. Therefore, a smooth transition is anticipated between the residue corridor and the projectile. The parametrization of this smooth transition constitutes the main task in designing the formula.

In a first step, a parameter set has been searched for that describes the fragmentation yields from projectiles located close to the line of $\beta$ stability. In a second step, a modification of the yield distributions due to the neutron or proton excess of projectiles located on the neutron- or proton-rich side of the line of $\beta$ stability (the “memory effect”) has been parametrized.

### B. Parameters of EPAX version 1

As explained in detail in Ref. [10], the cross section (in barns) of a fragment with mass $A$ and charge $Z$ produced by projectile fragmentation from a projectile $(A_p, Z_p)$ impinging on a target $(A_t, Z_t)$ is written as

$$\sigma(A,Z) = Y_A \sigma(Z_{\text{prob}} - Z) = Y_A n \exp(-R|Z_{\text{prob}} - Z|^{U_s(p)}).$$  

The first term $Y_A$ represents the mass yield, i.e., the sum of the isobaric cross sections with fragment mass $A$. The second term describes the “charge dispersion,” the distribution of elemental cross sections with a given mass around its maximum, $Z_{\text{prob}}$. The shape of the charge dispersion is controlled by the width parameter $R$ and the exponent $U_n$, $(U_p)$ on the neutron- (proton-) rich side of the residue corridor. The factor $n = \sqrt{R/\pi}$ simply serves to normalize the integral of the charge dispersion to unity.

The mass-yield curve is taken to be an exponential as a function of $A_p - A$. The slope of this exponential, $P$, is a function of the projectile mass. An overall scaling factor $S$ accounts for the peripheral nature of fragmentation reactions and therefore depends on the circumference of the colliding nuclei:

$$Y_A = S P \exp[-P(A_p - A)].$$  

$$S = S_2(A_p^{1/3} + A_t^{1/3} + S_1)[\text{b}],$$

$$\ln P = P_2 A_p + P_1.$$  

The numerical values of the various constants can be found in Table I.

The charge dispersion is characterized by the three parameters $R$, $Z_{\text{prob}}$, and $U$. These three parameters are strongly correlated and difficult to obtain uniquely with a least-squares fitting technique. Note that the isobar distributions are not symmetric on the neutron- and proton-rich sides; therefore $U$ has two different values $U_p$ and $U_n$ on the
proton- or neutron-rich side of the valley of \( \beta \) stability, respectively. In Ref. \([10]\), the exponent \( U \) for the neutron-rich side of the isobar distribution was chosen as \( U_n = 1.5 \), whereas the proton-rich side falls off like a Gaussian \( (U_p = 2) \).

The maxima of the charge dispersions, \( Z_{prob} \), have been parametrized relative to the valley of \( \beta \) stability, \( Z_{prob} = Z_{\beta} + \Delta \).

\[
Z_{\beta} = A / (1.98 + 0.0155A^{2/3}).
\]

\( \Delta \) is found to be a linear function of the fragment mass, \( A \), for heavy fragments \( (A \gg \Delta_4) \), and is extrapolated quadratically to zero:

\[
\Delta = \begin{cases} 
\Delta_3 A^2 & \text{if } A < \Delta_4, \\
\Delta_2 A + \Delta_1 & \text{if } A \geq \Delta_4.
\end{cases}
\]

Similar to the parameter \( Z_{prob} \) just discussed, the width parameter \( R \) is a function of fragment mass only, irrespective of the projectile. In Ref. \([10]\) it was found that the experimental \( R \) values can be approximated by an exponential of the form

\[
\ln R = R_2 A + R_1.
\]

The equations given above are sufficient to describe the ‘residue corridor,’ i.e., the yield distributions of projectiles located on the line of \( \beta \) stability if the fragment mass is far from the projectile mass. Close to the projectile, the following modifications have to be introduced \([10]\):

\[
\Delta = \Delta [1 + d_1 (A/A_p - d_2)^2],
\]

\[
R = R [1 + r_1 (A/A_p - r_2)^2].
\]

This serves to gradually reduce to zero the offset of \( Z_{prob} \) from the line of \( \beta \) stability and to decrease the width of the charge dispersion when \( A \) approaches the mass of the projectile, \( A_p \).

A final correction applies if the projectile does not lie on the line of \( \beta \) stability. In this case, the \( A/Z \) ratio of the fragments will to some extent ‘remember’ the \( A_p/Z_p \) ratio of the projectile (‘memory effect’). For an analytical de-
FIG. 2. Experimental mass yields $Y_A$ for the reactions of $^{40}$Ar + $^{12}$C [(a), Ref. [18]], $^{86}$Kr + $^8$Be [(b), Ref. [5]], $^{129}$Xe + $^{27}$Al [(c)], Ref. [6], and $^{208}$Pb + $^{64}$Cu [(d), Ref. [7]] compared to the old EPAX parametrization version 1 (dashed line) and to the new EPAX formula version 2 (solid line). The open squares are the sum of the experimental cross sections for a given fragment mass $A$, whereas the open triangles represent the calculated cross sections using the new EPAX formula for isotopes missing in the experimental data for a given mass $A$. The solid squares are the sum of the two values.

FIG. 3. Experimental charge-changing cross sections for $^{132}$Xe (a) and $^{197}$Au ions (b) on $^{27}$Al targets in comparison with the old EPAX parametrization version 1 (dashed line) and the new EPAX formula version 2 (solid line). The experimental data are from Binns et al. [19].

too large maxima. Moreover, the cross sections for fragments close to the projectile (with masses $A \geq 200$) are predicted much too high. Minor discrepancies were also found for $^{129}$Xe-fragment yields [6] where the experimental distributions were found to be wider than the EPAX predictions. Serious deficiencies were revealed when, compared to the EPAX prediction, too large cross sections for neutron-deficient isotopes from $^{58}$Ni fragmentation [4] and too small cross sections of neutron-deficient Sn isotopes from $^{112}$Sn fragmentation [16] were measured.

III. MODIFICATIONS OF THE EPAX FORMULA

A. Fragmentation of projectiles close to $\beta$ stability

The availability of extensive cross-section measurements for projectile fragmentation of $^{40}$Ar, $^{58}$Ni, $^{86}$Kr, $^{129}$Xe, and $^{208}$Pb [7,4–7] down to $\mu b$ or nb cross sections allowed in a first step to find a better parametrization of the mass yields $Y_A$. In a second step, we adjusted the parameters describing the residue corridor, i.e., the width parameter $K$ and the slope constants $U_n$ and $U_p$ of the quasi-Gaussian charge dispersion together with its centroid $Z_{prob}$. In a third step, the correction factors for isotopic yields close to the projectile were modified. Finally, modifications for projectiles outside

...
the valley of β stability were redetermined. Numerical values for the new constants are given in the fourth column of Table I.

1. Integrated mass and charge yields

According to Eqs. (2)–(4), \( Y_A \) is described by an exponential function depending on the fragment mass \( A \), the projectile mass \( A_p \), and the target mass, \( A_t \). In EPAX version 1, no additional correction close to the projectile has been performed. As shown by the dashed lines in Fig. 2, this yields only a very rough agreement with experimental data. Note that the experimental mass yields had to be complemented by calculated cross sections from EPAX version 2 where experimental data points were missing. For the \(^{40}\)Ar, \(^{86}\)Kr, and \(^{129}\)Xe data, the additional calculated cross sections contribute only close to the projectile; in the \(^{208}\)Pb case, however, the experimental data are less complete and more significant corrections were necessary.

Figure 2 shows that the slope somewhat further away from the projectile is reasonably well reproduced; however, the absolute height as well as the slope close to the projectile mass needed to be modified. A much better overall normalization was achieved by reducing the scaling factor \( S_2 \) by a factor of 0.6, as demonstrated by the solid lines in Fig. 2. An additional correction introduced in the new EPAX version consists of an increase in the mass yield \( Y_A \) close to the projectile according to

\[
Y_A = Y_A [1 + y_1 (A/A_p - y_2)]^2
\]  

for \( A/A_p \approx y_2 \). These modifications give a satisfactory agreement with experimental data. In the case of \(^{129}\)Xe, the slope far from the projectile mass deserves further attention. However, because of the lack of other complete experimental data, we were not able to significantly improve the mass-yield description.

Another comparison of integrated cross sections from EPAX to experimental data can be performed for charge-changing cross sections. For this purpose, we use experimental data from Binns et al. [19]. Figure 3 shows a comparison of the experimental data to the EPAX versions 1 and 2. As can be seen, nice agreement is achieved with the new EPAX formula, in particular close to the target. For the \(^{129}\)Xe data, the excess charge-changing cross sections for lower \( Z \) are most likely due to secondary reactions in the targets which are insufficiently accounted for.
2. Parameters of the residue corridor

In the upper part of Fig. 4, we show, as a function of the fragment mass $A$, the new position $Z_{\text{prob}}(A)$ of the “residue corridor,” for $^{129}$Xe and $^{208}$Pb, fragmentation. Data from $^{238}$U fragmentation [20] are also included in the figure; they were not used to fit the parameters $A_1$ and can serve as a check. For clarity, we plot the offset, $\Delta$, from $Z_p$ according to Eq. (5). The dashed lines are for EPAX version 1; the solid lines represent the new version 2. Note that for better results close to the projectile, we have added to $Z_{\text{prob}}$, according to Eq. (5) a tiny fragment-mass dependent correction amounting to 0.002$A$. The lines sloping downward above $A \approx 110$ and $A \approx 180$ illustrate the return of $Z_{\text{prob}}$ to $\beta$ stability close to the $^{129}$Xe and $^{208}$Pb projectiles according to Eq. (9).

In a similar way, the lower part of Fig. 4 visualizes the analytical description of the width parameter, $R$. While the functional form of the correction for $\Delta$, Eq. (9), was kept identical with EPAX version 1, the correction for $R$ near the projectile is now written as

$$R = R[1 + r_1 A_2 (A/A_p - r_2)^4] \quad (14)$$

for $A/A_p \geq r_2$, which yields more narrow charge distributions for $A$ close to $A_p$.

Only minor changes were introduced for the slope constants $U_a$ and $U_p$ of the neutron- and proton-rich sides of the charge distributions, respectively. While $U_a$ was increased from 1.5 to 1.65 (constant for all $A$), $U_p$ is now slightly fragment-mass dependent with

$$U_p = U_1 + U_2 A + U_3 A^2, \quad (15)$$

which approaches $U_p = 2.2$ for large $A$, compared to a previous constant value of 2.0 (Gaussian curve).

A close inspection of the width parameter $R$ resulting from fits to the measured $^{86}$Kr isotope distributions [5] shows that they are slightly but systematically decreased compared to Eq. (8) (see Fig. 4). This means that the isobar distributions are slightly wider for a neutron-rich projectile like $^{86}$Kr than for a $\beta$-stable one. Therefore, we tentatively introduce the following neutron-excess dependence of $R$:

$$R' = R[1 - 0.0833(Z_{p-} - Z_p)], \quad (16)$$

where $R$ is calculated according to Eq. (8). For neutron-deficient projectiles, $R$ is not changed.

For a typical example, the combined effect of the changes described above is visualized in Fig. 5, which displays the isotope distributions from 800$A$ MeV $^{129}$Xe fragmentation measured over four orders of magnitude [6]. In particular, the data measured near the maximum are now much better reproduced. The stronger increase in $R$ near the projectile mass, $A_p = 129$, leads to a much faster decrease of the corresponding fragment yields at $A \approx 124$. Similar observations, though over a smaller vertical range, can be made for the fragment distributions from $^{208}$Pb fragmentation shown in Fig. 1. It is interesting to note that even for $^{238}$U fragmentation products in the mass range $100 \leq A \leq 130$, produced in violent collisions and correspondingly long chains of statis-
tical evaporation, rather good agreement between experimental data [20] and EPAX can be observed.

**B. Fragmentation of neutron- or proton-rich projectiles**

In the previous EPAX version [10], the parametrization of the ‘‘memory effect’’ was fitted, for the case of neutron-rich projectiles, to the results from $^{48}$Ca fragmentation at 213AMeV [2] [see Eq. (12)]. For the case of $^{86}$Kr, we were able to check the validity of this prescription in a different fragment-mass range [5]. We have kept this prescription for the current version 2, since the agreement with experimental data has not changed significantly (see Fig. 6 for selected examples).

The situation is different, however, for the case of neutron-deficient projectiles. There, the previous version was unsatisfactory from the beginning: First, there were only very few radiochemical cross sections available to derive a guess for the memory effect on the neutron-deficient side. Second, the resulting formula $\Delta_m = [p_2(A/A_p)](Z_p - Z_{bp})$ does not yield the limiting value $\Delta_m/(Z_p - Z_{bp}) = 1$ for $A = A_p$. These deficiencies became particularly obvious when Schneider et al. [16] measured only small cross sections for neutron-removal products from $^{112}$Sn. This lead to a different functional form for the memory effect for neutron-deficient projectiles. The new parametrization reads

$$\Delta_m = \exp[p_1 + p_2 A/A_p](Z_p - Z_{bp}),$$

which is equivalent to a stronger but more rapidly decaying memory effect compared to the polynomial [Eq. (12)] used previously. The consequences of the new parametrization of the memory effect for neutron-deficient projectiles are visualized in Fig. 7. The small measured cross sections for $^{100}$Sn [16] are now better reproduced, while the good agreement (over more than seven orders of magnitude) with data measured in $^{124}$Xe fragmentation [21] is maintained.

We note in passing that Eq. (17) is very similar to what has been suggested by von Egidy and Schmidt [22] to reproduce production cross sections from antiproton annihilation. The overall agreement with the experimental fragmentation yields studied in our work is worse, however, when we adopt their description of the memory effect, in particular on the neutron-rich side. Moreover, we do not fit any set of parameters separately for a specific projectile as has been done in Ref. [22], but rather want to describe all systems with the same parameter set.

The yield distributions from the neutron-deficient projectile $^{58}$Ni, studied in great detail by Blank et al. [4], deserve special attention. Not only do they represent a rare example of cross-section measurements down to the subnambarn level, but also a case where severe discrepancies with EPAX version 1 were observed: the yields of even-Z isotopes close to the proton drip line where found to be enhanced in experiment by factors of up to 750 [4]. As a remedy for this deficiency, we have chosen to switch over from quasi-Gaussian to exponential slopes of the charge distribution above a certain gradient of the cross-section distribution. In order to achieve a steady transition between the two slopes, the transition point and the slope of the exponential have to be adjusted carefully. For this purpose, we calculate the derivative of the logarithm of the cross section [Eq. (1)]:

$$\frac{dF}{dZ} = \frac{d(\ln(\sigma))}{dZ} \approx -\frac{Z - Z_{prob}}{\ln(10)},$$

The transition point to the exponential slope, $Z_{exp}$, can be calculated for the proton-rich side as a function of the fragment mass $A$ according to

$$Z_{exp}(A) = Z_{prob}(A) + \frac{dF}{dZ} \int_{A}^{\infty} \frac{\ln(10)}{2R(A)}. \tag{19}$$

From $Z_{exp}$ on, the slope is exponential with the same gradient as Eq. (1) at this point.

The gradient for which we switch to the exponential slope has to be parametrized as a function of the fragment mass $A$. As for the moment only the $^{58}$Ni data exhibit this exponential trend (the measured cross sections in the $^{100}$Sn region reach the same cross section level, but are farther away from the proton drip line), we tried to adjust the function $dF/dZ$ in such a way to reproduce the $^{58}$Ni data without deteriorating the good agreement with the measured $^{124}$Xe data. The function which fulfills reasonably well these criteria is the following:

$$dF \approx \frac{1}{2R(A)} = 1.2 + 0.647(A/2)^{0.3}. \tag{20}$$

The result for the $^{58}$Ni fragment-yield distributions is visualized in Fig. 8. The solid line shows the new EPAX cross sections including the above modification of the slopes, whereas the omission of the exponential slope leads to the dotted cross-section curve. As a consequence of this modification, EPAX version 2 predicts a production cross section, e.g., for the doubly magic nucleus $^{48}$Ni of $4 \times 10^{-13}$ b, which is within reach of an experiment.

As stated earlier, the present adjustment for very proton-rich projectile fragments is only based on results from $^{58}$Ni fragmentation [4]. Therefore, caution is advisable when applying this parametrization especially to very light proton-rich fragmentation products. It would be interesting to compare the present parametrization to fragmentation yields at the drip line from other proton-rich beams, e.g., from $^{36}$Ar or $^{78}$Kr. In the $^{100}$Sn region, the present transition to an exponential slope has only a slight influence on the $^{100}$Sn cross-section prediction from $^{112}$Sn fragmentation (6.6$ \times 10^{-12}$ b, to be compared to an experimental value of 1.2$ \times 10^{-12}$ b [16]). It might be interesting to measure fragmentation cross sections for even more proton-rich nuclei in this mass region to compare to the modified EPAX formula.

**C. Overall quality of EPAX version 2**

Besides the comparison between experimental data and EPAX predictions for individual elemental distributions, we tried also to visualize in a more global manner the overall quality of the new EPAX parametrization compared to the data and the improvements with respect to EPAX version 1.
For this purpose, we plot the cross section ratios $\sigma_{\text{exp}}/\sigma_{\text{EPAX}}$ for all projectile-target combinations used in the present paper in Fig. 9. For most of the projectiles the logarithm of the cross-section ratio is centered around zero, indicating an agreement on average between experiment and prediction. For the new EPAX version 2, the sigma widths of these distributions vary between 0.5 in the case of $^{40}$Ar and $^{86}$Kr) and 0.2 in the case of $^{129}$Xe). If we analyze all experimental data together, we obtain a sigma of 0.4. This demonstrates that the new EPAX formula can predict cross sections in most cases within a factor of 2. In almost all cases we observe a significantly smaller sigma for the new EPAX parametrization than for the old version. This is particularly striking in the cases of $^{58}$Ni, $^{129}$Xe, and $^{208}$Pb. Only in the case of $^{48}$Ca does the agreement between experimental data and predictions deteriorate slightly, exhibiting now a shift to smaller predicted cross sections. It is likely that the memory effect for neutron-rich projectiles is the origin of this discrepancy.

D. Comparison with other empirical parametrizations

As has been mentioned in the Introduction and discussed in detail in Ref. [10], other empirical parametrizations have been proposed, mainly for proton-induced spallation reactions (e.g., Refs. [23,24]). Contrary to our formula, these parametrizations fit also the energy dependence of the cross sections. The former approach has been extended to describe heavy-ion induced spallation reactions by scaling proton-induced cross sections by an energy-dependent factor [25] and has achieved good agreement with measured cross sections (in the 100 mb to 1 mb range) for fragments from medium-mass nuclei (i.e., for projectiles up to $^{56}$Fe [25]).

For fragmentation cross sections from heavy-mass nuclei, however, the formula of Tsao et al. [25] is less successful. This can be demonstrated, e.g., by comparing in Fig. 10 their prediction for the Pt isotope distribution in the reaction $^{208}$Pb+$^{nat}$Cu to experimental data [7]. As we have mentioned in the Introduction, our physical understanding of high-energy heavy-ion reactions suggests that proton-induced reactions produce more neutron-rich isotope distributions of heavy elements than heavy-ion-induced reactions; therefore it is unlikely that even a better fit of the formula of Tsao et al. to the data in Fig. 10 would apply also to the same isotope distribution formed in the $p+^{208}$Pb reaction.

We believe that separate parameter sets have to be fitted to the respective experimental data. For $\approx 1$A GeV protons im-
ping on heavy targets, the recent work of Refs. [13,14] provides for the first time a comprehensive data set that allows to extend previous work [23,24] to heavier fragments. A complete parametrization of the bombarding-energy dependence, however, has to await more measurements at different energies for the heavy systems.

E. EPAX predictions for projectiles very far from stability

The parameters of the new EPAX formula have been adjusted by fitting data from stable projectiles. As explained above, one of the most difficult tasks of the present work was to reasonably parametrize the cross sections for fragments close to the projectile. If the projectile is very far from the line of $^{b}$ stability, this task is even more difficult.

The comparisons between EPAX predictions and experimental data shown in the present paper indicate that the chosen parametrization is reasonable. A possible check of its predictive power could be to compare the EPAX formula to other data from neutron-rich or neutron-deficient projectiles. However, only very few other high-quality data are available.

As one check, we compare in Fig. 11 the fragmentation of a stable $^{24}$Mg beam [18] to a radioactive $^{28}$Mg beam [26] and of a stable $^{40}$Ar beam [1] to a radioactive $^{43}$Ar beam [26]. The elemental distributions for Mg, Na, and Ne isotopes shown in the left-hand column and those for S, P, and Si isotopes shown in the right-hand column nicely show the experimental memory effect and the high quality of the EPAX predictions. In a similar way we compared the results from fragmentation of $^{96}$Ru and $^{96}$Zr [27] to EPAX predictions and found reasonable agreement. However, for projectile beams still further away from stability, the memory effect becomes more and more important. Therefore, we think that some caution should be applied when using EPAX predictions for projectiles very far from stability like, e.g., $^{132}$Sn. Here, no experimental data at all exist to verify the EPAX predictions.

IV. SUMMARY AND CONCLUSIONS

We have demonstrated that the quality with which EPAX reproduces measured high-energy fragmentation cross sec-

FIG. 10. Platinum ($Z=78$) isotope distribution from 1A GeV $^{208}$Pb fragmentation in a $\mathrm{nat}$Cu target [7] in comparison with the new EPAX parametrization (solid curve) and the formula of Tsao et al. (Ref. [25], dashed curve).

FIG. 11. Experimental cross sections for the fragmentation of stable (open symbols) and radioactive beams (solid symbols) compared to the new EPAX parametrization (dashed and solid lines, respectively). On the left-hand side, results from $^{24}$Mg [18] and $^{28}$Mg [26] fragmentation are shown, whereas on the right-hand side $^{40}$Ar [1] fragmentation is compared to $^{43}$Ar [26] fragmentation.
this is the difference in experimental cross sections for $^{100}$Sn production from a $^{112}$Sn beam of roughly a factor of 100 in recent GANIL and GSI experiments [21,28]. In addition, these intermediate-energy data are measured with separators having very low transmissions (of the order of a few percent) which are difficult to measure experimentally. It would be interesting, however, to compare EPAX also to cross sections obtained at lower incident energies once high-quality data become available.

A FORTRAN program of the EPAX formula as well as additional graphs comparing experimental data to EPAX predictions can be downloaded from the following address: ftp://ftpccbg.in2p3.fr/pub/nex/epax

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